Reliability Analysis of a Tunnel Design with RELY

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Summary

RELY is a novel SOFiSTiK module for reliability analysis. It allows to employ the full capabilities of the SOFiSTiK finite element package to model the engineering system of interest, in combination with a powerful stochastic model and analysis toolbox. The kernel of RELY is powered by Strurel, one of the leading reliability software tools.

We show how to apply RELY to perform reliability analysis of a conventionally driven tunnel. Model parameters that are subject to considerable uncertainty are represented stochastically. The first-order reliability method (FORM) is employed to estimate the reliability of the outer tunnel lining. Sensitivity information provided by FORM is used to quantify the relative importance of the stochastic model parameters.

Zusammenfassung

RELY ist ein SOFiSTiK Modul zur Zuverlässigkeitsanalyse, das mit der Version SOFiSTiK 2016 eingeführt wurde. RELY ermöglicht die probabilistische Analyse komplexer mit SOFiSTiK erzeugter Finite Element Modelle. Der Rechenkern von RELY basiert auf dem Rechenkern von Sturel, einer führenden Software zur Zuverlässigkeitsanalyse.

Wir benutzen RELY zur Zuverlässigkeitsanalyse eines mit der Neuen Österreichischen Tunnelbaumethode vorgetriebenen Tunnels. Modellparameter die nennenswerten Unsicherheiten unterliegen, werden durch Wahrscheinlichkeitsverteilungen beschrieben. Die Zuverlässigkeit der mit Spritzbeton hergestellten Außenschale des Tunnels wird mit der First-Order Reliability Method (FORM) untersucht. Darüber hinaus werden mit dem FORM Verfahren erhaltene Informationen zur Sensitivität der stochastischen Parameter genutzt, um die relative Wichtigkeit der stochastischen Modellparameter zu beschreiben.

1 Reliability-based structural design and assessment

An engineering structure must be adequately stable and utilizable during its lifetime. It is the responsibility of the engineer to design the structure such that the safety requirements are met. Traditionally, compliance of an engineering system is demonstrated by means of safety factors. The safety concept for civil engineering structures is specified in Eurocode 0. The Eurocodes are primarily based on the concept of partial safety factors. An alternative to this classical approach is specified in Appendix C of Eurocode 0: By means of structural reliability techniques, the safety of an engineering system can be assessed probabilistically. The following is a brief summary of the main concepts for assessing the design of structures. A more detailed introduction can be found in (Straub 2015).

1.1 Design of structures using partial safety factors

In the partial factor method, partial safety factors and combination factors are used to evaluate a design value for demand E_d and capacity R_d. The design is considered sufficiently reliable if $R_\mathrm{d} \geq E_\mathrm{d}$. The design based on partial safety factors is usually a cost-effective design concept for most practical problems.

We illustrate the partial factor method by means of a rather simple example (This example is also used in the next section to explain the design based on probabilistic techniques): The task is to determine the diameter d of a rod under tension in order to maintain structural safety. The material of the rod to design is steel S235. The characteristic value for the yield stress f_y of steel S235 is $235\mathrm{N/mm}^2$. The characteristic value R_k for the capacity of the rod depends on the diameter d and is: $R_\mathrm{k} = f_y \cdot 0.25 \cdot \pi \cdot d^2$. The partial safety factor used to obtain the design value for capacity R_d is $\gamma_R = 1.1$ (Eurocode 3). Thus, the design value for capacity is $R_\mathrm{d} = R_\mathrm{k}/\gamma_R$. The lower end of the rod is loaded permanently with one cubic meter of oak wood. According to EN 1990, Section 4.1.2, the characteristic value of a permanent load can be selected as the 95% quantile. The average value and coefficient of variation for the weight of oak wood is given in the JCSS Probabilistic Model Code: the average is $6\mathrm{kN/m}^3$ and the coefficient of variation is 10%. The mean value and coefficient of variation of the applied load is consequently $\mu_\mathrm{E} = 6\mathrm{kN}$ and $\delta_\mathrm{E} = 10\%$. It is appropriate to assume that the load follows a Normal distribution. The characteristic value of the demand is the 95% quantile of its distribution. Because the load follows the Normal distribution, we have $E_\mathrm{k} = \mu_\mathrm{E} \cdot (1 + k_E \delta_\mathrm{E})$, where $k_E = 1.64$; i.e., $E_\mathrm{k} = 7\mathrm{kN}$. The design value for the demand is defined in terms of E_k as $E_\mathrm{d} = E_\mathrm{k} \cdot \gamma_E$, where for permanent loads a partial safety factor of $\gamma_E = 1.35$ is required by the code. To ensure a sufficiently reliable design, the design value R_d of the capacity must be at least as large as E_d. Based on this condition, we can compute the required characteristic value for the resistance as: $R_\mathrm{k} \geq E_\mathrm{k} \cdot \gamma_E \cdot \gamma_R$. Consequently, the required minimum diameter d_min is:

$$d_\mathrm{min} = \sqrt{\frac{E_\mathrm{k} \cdot \gamma_E \cdot \gamma_R}{f_y \cdot 0.25 \cdot \pi}} = \sqrt{\frac{7\mathrm{kN} \cdot 1.35 \cdot 1.1}{235 \cdot 10^3 \mathrm{kN/m}^2 \cdot 0.25 \cdot \pi}} = 7.5\mathrm{mm}$$

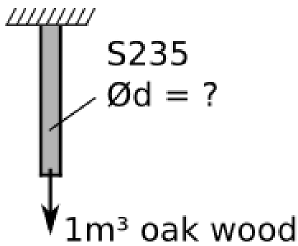

Fig. 1: A tension rod is loaded with 1m³ of oak wood. The diameter of the rod that is required for a safe design is to be determined. This simple example is tackled with the classical design approach based on safety factors (Section 1.1) and by means of a probabilistic design approach (Section 1.2).

1.2 Design of structures based on probabilistic techniques

Appendix C of Eurocode 0 specifies the design of structures based on probabilistic techniques. Such a probabilistic design is usually more complex than the design based on partial safety factors and, therefore, cost-effective only in special cases. In a probabilistic design, uncertainties in loading, material properties and geometry are taken into account explicitly; i.e., the uncertainties are represented by means of probability distributions. The target quantity of interest in reliability analysis is the probability of failure p_f of the investigated structure. Note that the obtained probability of failure should not be interpreted as the probability that the investigated structure will actually fail: For example, human errors can have a considerably impact on the reliability of a structure, but they cannot be properly accounted for in the reliability analysis. The probability of failure p_f obtained by means of structural reliability is used to compare the level of safety of different structures quantitatively.

The probabilities of failure p_f of engineering structures are commonly small; i.e., $p_f \ll 10^{-3}$. As a consequence, it is often clearer to communicate the reliability of a structure by means of the so-called reliability index β. The reliability index is defined as $\beta = -\Phi^{-1}(p_f)$, where $\Phi^{-1}(\cdot)$ is the function of the cumulative distribution function of the standard Normal distribution. The reliability index increases if the reliability of the investigated structure is increased and the corresponding probability of failure is decreased. The method of partial factors should ensure the reliability of the structure to be at least $\beta \geq 3.8$ within a reference period of 50 years (this corresponds to $p_f \leq 7 \cdot 10^{-5}$), according to Annex B in Eurocode 0. Annex B in Eurocode 0 distinguishes three consequence classes. For increasing consequences, the required reliability index is increased. The requirement for a reliability index of at least 3.8 refers to the second class of consequences and a 50-year reference period.

We will use the example already presented in the previous section to illustrate the probabilistic design of a structure. Our aim is to evaluate the reliability index for the previously computed diameter $d_{\min} = 7.5\text{mm}$. The probability of structural failure can be expressed as the probability that the demand E (the load that acts on the structure) exceeds the capacity R of the structure; i.e., $\Pr(R - E < 0)$. The mean and coefficient of variation of E is given in the last section as $\mu_{\mathrm{E}} = 6\text{kN}$ and $\delta_{\mathrm{E}} = 10\%$. The coefficient of variation δ_y of the yield stress of steel is selected as 7% based on the JCSS Probabilistic Model Code, Part 3. We assume that the yield stress y follows a Normal distribution. Considering that the coefficient of variation of 7% is small, this assumption is justifiable – as the probability of negative values is practically zero. The characteristic value of the yield stress f_y corresponds to the 5% quantile of y; hence, $k_y = -1.64$ (the quantity k_y is defined as the inverse of the cumulative distribution function of the standard normal distribution, evaluated at 5%). Consequently, the mean value of the yield stress μ_y can be obtained as $\mu_y = f_y/(1 + k_y\delta_y) = 235\text{N/mm}^2/(1 - 1.64 \cdot 0.07) = 265\text{N/mm}^2$. The capacity of the tension rod is $R = y \cdot 0.25 \cdot \pi \cdot d^2$ and follows a Normal distribution as well. The mean μ_{R} and coefficient of variation δ_{R} of R is $\mu_{\mathrm{R}} = 11.7\text{kN}$ and $\delta_{\mathrm{R}} = 7\%$. Let the difference between the capacity and load be denoted as $M = R - E$. As both R and E follow a Normal distribution, M is also Normal. The expectation of M is $\mu_M = \mu_R - \mu_E = 5.7\text{kN}$, and the standard deviation of M is $\sigma_M = \sqrt{\sigma_R^2 + \sigma_E^2} = 1.0\text{kN}$. As M follows a Normal distribution, the reliability index can be calculated explicitly as:

$$\beta = \frac{\mu_M}{\sigma_M} = 5.7$$

The evaluated reliability index of 5.7 is larger than any reliability index demanded in Annex B of Eurocode 0. Therefore, the design with $d_{\min} = 7.5\text{mm}$ can be regarded as comparatively conservative. However, this finding holds only for the test case at hand and should not be generalized.

1.3 Limit-state function

The limit-state function g is a function that is by definition negative in case of failure and positive otherwise. If resistance R and action E can be clearly separated, the limit-state function is usually defined as $g = R - E$. For some systems a separation of demand and capacity is not feasible (e.g., due to soil-structure interaction in tunneling). If demand and capacity cannot be separated easily, the limit-state function is often defined as the difference between some threshold value and the corresponding model output; e.g., the displacement at the tip of a cantilever beam versus the maximum allowed displacement for that system.

1.4 The reliability problem

Based on the definition of the limit-state function, the probability of failure p_f can be expressed as:

$$p_f = \int_{g(\mathbf{x})\leq 0} p_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x}$$

where $\mathbf{X}$ is an M-dimensional vector of uncertain input quantities of the system of interest, and $p_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of $\mathbf{X}$. The above integral can often not be evaluated analytically, because the domain $\{g(\mathbf{x}) \leq 0\}$ is not known explicitly. Instead, the integral is usually solved numerically. The probabilities that we are dealing with in reliability analysis are typically rather small; i.e., $p_f \ll 10^{-2}$. This renders the numerical treatment of the reliability integral difficult, because $\{g(\mathbf{x}) \leq 0\}$ constitutes only a small part of the total domain of $\mathbf{X}$.

1.5 Commonly used reliability methods

The class of numerical methods specifically designed to solve the reliability integral introduced in the previous section are referred to as *reliability methods*. The various reliability methods differ in their treatment of the reliability integral.

The most straight-forward (and simplest) method to solve the reliability problem is *Monte Carlo Simulation* (MCS). However, for small failure probabilities MCS requires a prohibitively large number of limit-state function evaluations. Hence, efficient reliability methods have been developed that aim at minimizing the number of required limit-state function calls. This is because in structural reliability, the limit-state function is commonly expressed as a function that depends on the outcome of a finite element analysis. Consequently, for every limit-state function evaluation, a finite element analysis must be performed – which renders the reliability analysis of large finite element systems computationally expensive.

Besides MCS, other well-known reliability methods are the *First Order Reliability Method* (FORM) (Hasofer and Lind 1974; Rackwitz and Flessler 1978), the *Second Order Reliability Method* (SORM) (Breitung 1984), *importance sampling methods* including *line sampling* (Hohenbichler and Rackwitz 1988; Koutsourelakis et al. 2004; Rackwitz 2001) and *directional importance sampling* (Bjerager 1988; Ditlevsen et al. 1990), and *Subset Simulation* (Au and Beck 2001; Papaioannou et al. 2015).

1.6 First Order Reliability Method (FORM)

FORM, which is explicitly referred to in Appendix C of Eurocode 0, computes an approximation of the probability of failure based on a linearization of the limit-state function. The random variable space $\mathbf{X}$ of the problem is transformed to an underlying space of independent standard Normal random variables, denoted $\mathbf{U}$. FORM linearizes the limit-state function with respect to $\mathbf{U}$ at the so-called design point $\mathbf{u}^*$. The design point $\mathbf{u}^*$ is defined as the point on the failure surface (i.e., the region where the limit-state function is zero) that is closest to the origin and represents the most likely failure point in the outcome space of $\mathbf{U}$. The principal idea behind FORM is illustrated in *Fig. 2*. The probability of failure is computed based on the linearized limit-state function around $\mathbf{u}^*$.

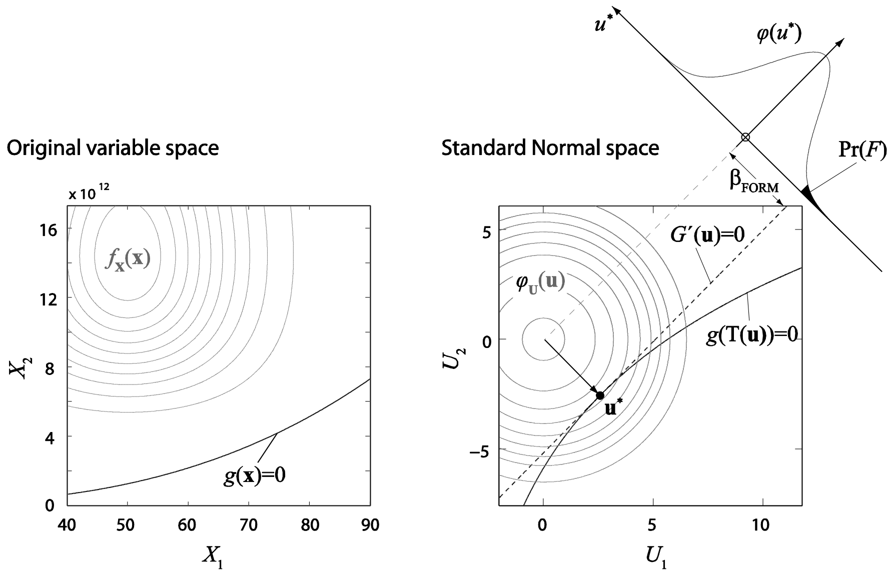

Fig. 2: Design point and linear approximation of the limit-state surface. Left-side: original random variable space; right-side: standard Normal space. [taken from (Klüppelberg et al. 2014)]

The design point $\mathbf{u}^*$ is obtained by solving a constraint optimization problem. The main computational burden in FORM is to solve this optimization problem. Since an optimization problem has to be solved, the limit-state function is usually required to be differentiable.

As a by-product of FORM, the random variables that have the largest influence on the total variance of the linearized limit-state function can be determined.

2 The SOFiSTiK module RELY

RELY is a module of SOFiSTiK that performs reliability analysis. The engineering system of interest is modeled using the full capabilities of the SOFiSTiK finite element package. The kernel of RELY is powered by the well-established reliability software Strurel. Numerous structural reliability methods are included: Monte Carlo simulation, FORM, SORM, importance sampling, line sampling, directional sampling, adaptive sampling and subset simulation. RELY is available in SOFiSTiK starting from version SOFiSTiK 2016.

The definition of failure criteria by means of limit-state functions is very flexible and powerful in SOFiSTiK. All relevant information about the engineering system can directly be read from the comprehensive SOFiSTiK database CDB.

3 Investigated engineering problem: reliability analysis of a tunnel

To reduce traffic congestion in the German town of Freising, a bypass west of the town is planned. The bypass involves the construction of a tunnel. The conventional driven part of the tunnel has a total length of 465m. The building authority of Freising is in charge of the planning of the tunnel. The tunnel is designed using a finite element model of the tunnel by the engineering office EDR GmbH. Based on this project, we exemplarily show how to investigate a tunnel probabilistically by means of reliability analysis. The goal is to assess the reliability of the shotcrete lining and to determine the model parameters that have the largest influence on the reliability of the tunnel.

The purpose of this investigation is to show how the SOFiSTiK module RELY can be applied to assess the reliability of a tunnel. The actual dimensioning of the tunnel lining – which is conducted by EDR GmbH – was done independently of this probabilistic analysis. We emphasize that the results obtained in this study should not be misinterpreted as the reliability associated with the mentioned tunneling project for the following reasons: 1) The probabilistic description of the input is based mainly on a geological survey (Vogt et al. 2015) and on engineering standards. If this were an actual consulting project, we would have to explicitly consult experts (i.e., geologists and material testers) for specific information regarding the uncertainties at hand. 2) We perform the reliability analysis only for a selected section of the tunnel (i.e., the section located 405m after the beginning of the tunnel). In the project at hand, groundwater levels and geotechnical conditions vary considerably throughout the tunnel. Therefore, for the analysis to be meaningful, we would have to analyze multiple tunnel sections. 3) In the present analysis, we assess the reliability of the shotcrete lining. An equally important query would be to assess tunnel induced settlements, as the tunnel passes beneath area covered by buildings (Camós et al. 2016).

4 Mechanical model of the tunnel

4.1 General

The investigated tunnel is a conventional driven tunnel with a tapering cross-section. The problem is modeled in the SOFiSTiK finite element (FE) software package (SOFiSTiK AG 2016), using 2D plane strain finite elements. The numerical model has a width of $60\mathrm{m}$ and a height of $40\mathrm{m}$. The FE mesh and the tunnel profile are illustrated in *Fig. 3*.

The tunnel is located in a depth of approximately $10\mathrm{m}$ below ground. The tunnel profile has a height of $10.5\mathrm{m}$ and a width of $12.5\mathrm{m}$. The shotcrete lining is modeled using linear elastic beam elements. The required thickness of the shotcrete lining is $0.3\mathrm{m}$.

The reliability of the tunnel lining is investigated for an exemplarily picked section of the tunnel. A load of 1.2 MN is distributed at ground level over a length of 10m to model the weight of an existing structure (see *Fig. 4*).

4.2 Excavation process

The excavation process is modeled by application of the stress reduction method (Swobota 1979, Panet and Guenot 1982, Schikora and Ostermeier 1988). In this method, a specified fraction α of the initial stress is left inside the excavated tunnel area. The remaining stresses act as a support pressure to approximately account for the three-dimensional arching effect. The support pressure is removed from the model after installation of the lining. The parameter α is referred to as relaxation factor.

The excavation and installation of the tunnel lining is modeled in five steps: 1) representation of primary soil stresses before tunnel construction, 2) relaxation of the top heading, 3) excavation of the calotte and installation of shotcrete, 4) relaxation of the stope and invert, 5) excavation of the stope and invert, and installation of shotcrete.

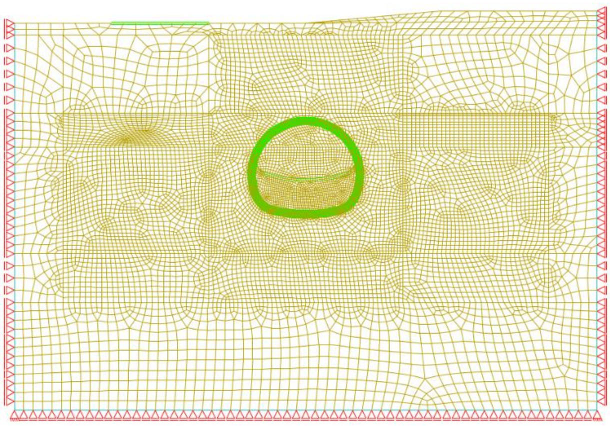

Fig. 3: Finite element mesh used to discretize the tunnel.

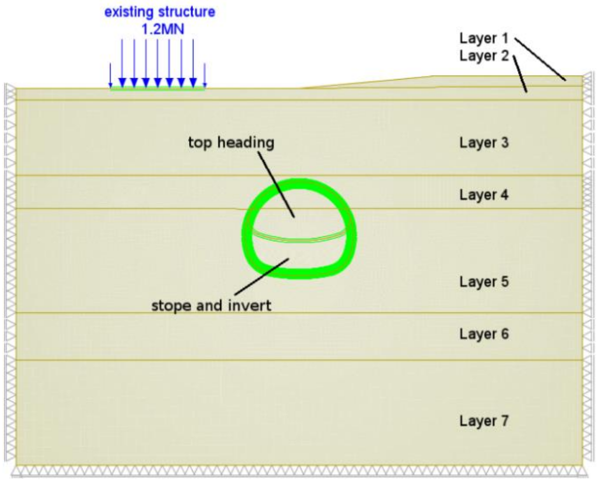

Fig. 4: Ground layers considered in the model.

4.3 Soil properties

The available geological report (Vogt et al. 2015) suggests layered soil conditions. Seven horizontal ground layers with a fixed (deterministic) height are considered in the model (see *Fig. 4*). The height of the seven soil layers is, for increasing depth: 1.1m, 1.2m, 8m, 3.5m, 11m, 5m and 11m.

A hardening plasticity soil model (SOFiSTiK AG 2016) is used to describe the material behavior of the individual soil layers. The relevant material properties of the soil layers as given in the geotechnical report (Vogt et al. 2015) are listed in *Table 1*. For some values of the coefficient for lateral earth pressure K_0 (see *Table 1*), a potential uncertainty is explicitly mentioned in (Vogt et al. 2015). The reference pressure p_{ref} and the exponent m of the material model are not listed in *Table 1*. The parameters p_{ref} and m describe the stress dependent stiffness of soil. According to (Vogt et al. 2015), p_{ref} and m are subject to large uncertainty, and a stochastic model is utilized for these material parameters, as presented in *Section 4*. The angle of dilatancy is assumed as zero, corresponding to a non-associated flow rule. Poisson's ratio is 0.35 for all soil layers (Vogt et al. 2015).

5 Stochastic model

The stochastic model is based upon the previously developed model, which was used in deterministic calculations by structural engineers at EDR. All parameters whose values are subject to notable uncertainty, are represented explicitly as random variables. The main uncertainties present in the problem at hand can be categorized into: uncertainties in the material parameters of the soil and the shotcrete, and model uncertainties. These uncertainties are discussed in the following.

layer	soil type	φ' [°]	c' [kN/m²]	γ [kN/m³]	E_S [MN/m²]	$E_{\mathrm{oed}}^{\mathrm{ref}}$ [MN/m²]	E_{50}^{ref} [MN/m²]	$E_{\mathrm{ur}}^{\mathrm{ref}}$ [MN/m²]	K_0
1	cover layer	20	5	20	5	5	5	10	1-sinφ'
2	tertiary clay/silt (soft)	25	5	20	7.5	7.5	7.5	20	1-sinφ'
3	tert. sand (med. compact)	35	0	21	50	50	50	110	1-sinφ'
4	tert. clay/silt (semisolid)	25	20	21	60	60	60	125	0.6–0.8
5	tertiary sand (compact)	37.5	0	22	120	120	120	250	0.4–0.6
6	tertiary gravel (compact)	37.5	0	22	120	120	120	250	0.4–0.6
7	tert. clay/silt (semisolid)	25	20	21	60	60	60	125	0.6–0.8

Table 1: Soil properties of the different layers according to the available geotechnical report (Vogt et al. 2015). The properties listed are friction angle φ', cohesion c', weight γ, Young's modulus for compressive loading $E_{\mathrm{oed}}^{\mathrm{ref}}$, Young's modulus for deviatoric stresses E_{50}^{ref}, Young's modulus for unloading and reloading $E_{\mathrm{ur}}^{\mathrm{ref}}$, and coefficient for lateral earth pressure K_0. The numbers highlighted in *red* indicate parameter uncertainties explicitly addressed in the report.

	$K_{0,4}$	$K_{0,5}$	$K_{0,6}$	$K_{0,7}$
$K_{0,4}$	1	0.4	0.2	0.1
$K_{0,5}$	0.4	1	0.4	0.2
$K_{0,6}$	0.2	0.4	1	0.4
$K_{0,7}$	0.1	0.2	0.4	1

Table 2: Correlation matrix for lateral earth pressure coefficients K_0.

5.1 Stochastic representation of the soil

The parameters in the soil layers that exhibit the largest uncertainty are p_{ref} and m (Vogt et al. 2015). The geotechnical report (Vogt et al. 2015) suggests, to investigate three different combinations of p_{ref} and m: [soft: $p_{\mathrm{ref}} = 300\mathrm{kN/m^2}$, $m = 0.4$], [medium: $p_{\mathrm{ref}} = 100\mathrm{kN/m^2}$, $m = 0.4$] and [stiff: $p_{\mathrm{ref}} = 100\mathrm{kN/m^2}$, $m = 0.6$]. Based on this information, we model p_{ref} as a lognormal distribution with mean 167 kN/m² and coefficient of variation 33%, and m is modeled as a lognormal distribution with mean 0.47 and coefficient of variation 11%. Individual soil layers are modeled as independent, i.e. correlation coefficients between parameters of different layers are taken as zero. The parameters p_{ref} and m of a single soil layer are assumed to be correlated with correlation coefficient -0.7; the correlation coefficient of -0.7 is deduced from the three combinations of p_{ref} and m listed in (Vogt et al. 2015). Technically, the exact values of $E_{\mathrm{oed}}^{\mathrm{ref}}$, E_{50}^{ref} and $E_{\mathrm{ur}}^{\mathrm{ref}}$ are not known with certainty. However, we assume that the three quantities are perfectly correlated and the parameter uncertainty about the stiffness of soil is already adequately represented by means of p_{ref} and m. Moreover, parameter uncertainty about the true value of φ', c' and γ is not considered in the analysis.

For soil layers 4 to 7, the coefficients of lateral earth pressure K_0 are regarded as uncertain in (Vogt et al. 2015) – see also *Table 1*. We model the values of K_0 by means of a lognormal distribution. The corresponding mean and standard deviation is chosen such that the bounds listed in *Table 1* correspond to the 5% and 95% quantile of the distribution. The assumed correlation coefficients between the individual K_0 are listed in *Table 2*.

5.2 Stochastic representation of the shotcrete

Shotcrete is a type of concrete that is projected with high velocity on the freshly excavated tunnel wall. Shotcrete is directly compacted when it hits the wall. Due to the particular construction process, the material properties of shotcrete are subject to larger uncertainties than the material properties of standard concrete. For example, the distance of the nozzle from the wall as well as the angle and velocity at which the shotcrete hits the wall considerably influence the material properties of shotcrete (Maidl 1992).

The most relevant material parameters of shotcrete with respect to the mechanical model at hand are its compressive strength and its Young's modulus.

Compressive strength: The applied shotcrete is required to meet the demanded strength of concrete C25/30 as specified in Eurocode 2. The average compressive strength of C25/30 measured on a defined concrete cylinder is given in Eurocode 2 as $f_{cm} = \mu_{f_{co}} = 33\mathrm{N/mm^2}$. The associated characteristic value is defined as the 5% quantile of the distribution of compressive strength and is specified as $f_{ck} = 25\mathrm{N/mm^2}$ for C25/30 (Eurocode 2). A lognormal distribution is suggested in Appendix C of Eurocode 0 for the modelling of uncertain material properties. Employing this distribution, the standard deviation of the compressive strength of C25/30 can be calculated as $\sigma_{f_{co}} = 5.34\mathrm{N/mm^2}$. The quantity f_{co} represents the uniaxial compressive strength of a concrete cylinder under well-defined (laboratory) conditions. The uncertainty in the in situ compressive strength f_c of the shotcrete lining is very likely larger than the uncertainty in f_{co}. Moreover, we assume that on average the in-situ f_c will be smaller than f_{co} which is obtained in a controlled environment, due to the numerous factors that have to be met in a laboratory experiment, but cannot always be controlled on-site. We express the relation between f_c and f_{co} as $f_c = f_{co} \cdot Y_c$, where Y_c is modeled as a lognormal random variable that is independent from f_{co} and has mean 0.8 and coefficient of variation 10%. As both f_{co} and Y_c are lognormal, the distribution of f_c is also lognormal with mean $\mu_{f_c} = 0.8 \cdot 33\mathrm{N/mm^2} = 26.4\mathrm{N/mm^2}$ and coefficient of variation $\delta_{f_c} = 19\%$. Note: The probabilistic description of f_c is mainly based on the recommendations given in the JCSS Probabilistic Model Code. During construction, the quality of shotcrete is constantly verified on-site. This means that in practice, the mean of Y_c is expected to be larger and the coefficient of variation smaller than assumed in this investigation. This is not considered in the chosen distribution for Y_c, and, therefore, the probabilistic description of f_c is conservative. By consulting an expert and/or assuring the quality on site, the uncertainties in Y_c could be more appropriately quantified, and the probabilistic description of Y_c could deviate from the recommendations given in the JCSS Probabilistic Model Code.

Young's modulus: The Young's modulus of concrete is closely related to the compressive strength of concrete. Eurocode 2 suggests to work with an average Young's modulus E_{cm} that is based on the average compressive strength f_{cm}: $E_{cm} = 11 \cdot 10^3 \cdot f_{cm}{}^{0.3}$. A similar relationship is employed in the JCSS Probabilistic Model Code to represent the uncertainties in the Young's modulus. We use a stochastic model to describe the Young's modulus as a random variable E_c that is based on both Eurocode 2 and the JCSS Probabilistic Model Code: $E_c = 11 \cdot 10^3 \cdot f_c{}^{0.3} \cdot Y_E$, where Y_E is a random variable that expresses the conditional uncertainty about E_c if f_c is known. We model Y_E as lognormal with mean 1.0 and coefficient of variation 15%, as recommended in the JCSS Probabilistic Model Code. Note that in our stochastic model, E_c is not modeled directly, but is expressed as the product of the two random variables f_c and Y_E. The marginal distribution of E_c can be obtained analytically; it is lognormal with mean $29 \cdot 10^3\mathrm{N/mm^2}$ and coefficient of variation 16%. Note: The early loading of the tunnel lining causes its effective stiffness to be smaller than the chosen probabilistic model of E_c suggests (Kusterle et al. 2014). This means that the mean of Y_E could in principle be reduced – the coefficient of variation of Y_E would be increased in this case. However, it is difficult to quantify the corresponding uncertainties, because the mechanical and chemical processes that lead to such a reduced stiffness are complex and cannot be modelled in a straightforward way. In the investigation at hand, this effect is not considered. As a larger stiffness of the tunnel lining leads to larger internal forces, the applied probabilistic model for the Young's modulus is on the conservative side.

5.3 Limit-state function

We analyze failure of the shotcrete lining. We describe failure as the event that admissible normal forces and bending moments are exceeded on at least one location of the shotcrete lining. This means that the overall limit-state function can be expressed as the minimum of all local limit-state functions – where local refers to a particular cross-section of the shotcrete lining.

name	group	type	mean	CV
f_{co}	concrete	lognormal	$33\mathrm{N/mm^2}$	16%
Y_c	concrete	lognormal	0.8	10%
Y_E	concrete	lognormal	1.0	15%
α	model	beta	0.4	10%
$p_{\mathrm{ref},i}$	soil	lognormal	167 kN/m²	33%
m_i	soil	lognormal	0.47	11%
$K_{0,4}$, $K_{0,7}$	soil	lognormal	0.68	10%
$K_{0,5}$, $K_{0,6}$	soil	lognormal	0.49	13%

Table 3: List of basic random variables employed in the problem. The name, type, mean and coefficient of variation (CV) is listed for each random variable. Note that for $p_{\mathrm{ref},i}$ and m_i, $i = 1, \ldots, 7$; and for K_0, $i = 4, \ldots, 7$. The stochastic model contains a total number of 22 random variables.

At a particular cross-section we take the normal force N and bending moment M in the shotcrete lining computed with the SOFiSTiK finite element model. We neglect that the shotcrete lining in the tunnel is reinforced (this assumption is discussed in the next section) and perform the structural analysis assuming the shotcrete lining as unreinforced. For the given N and M we compute the eccentricity e of the normal force N that generates bending moment M. For the obtained eccentricity e we determine the maximum normal force N_R that the cross-section can withstand and compare it to N. If $N_R \leq N$, failure occurs. Consequently, the local limit-state function of the corresponding cross-section can be expressed as $g_l = N_R - N$.

In order to determine N_R for a particular eccentricity e, the stress-strain curve of concrete is required. The tensile strength of concrete is assumed to be zero. Furthermore, we assume that the theory of Bernoulli applies (i.e., strains are linear over the cross-section). Thus, the normal force that the cross-section can withstand is the integral of the compressive strength as a function of strain according to the stress-strain curve over the compressed area of the cross-section. The value of N_R is obtained by selecting linear strains of the cross-section such that the generated normal force is maximized for the given eccentricity e.

The selected shape of the stress-strain curve of concrete is of particular relevance in the absence of safety factors. For example, the bi-linear stress-strain curve that can be applied according to Eurocode 2 is only conservative as long as the factor of safety associated with the allowable compressive strength is chosen large enough. For a factor of safety of one, the bi-linear stress-strain curve does not give conservative values for N_R compared to the non-linear stress-strain curve suggested in section 3.1.5 of Eurocode 2. As a consequence, we evaluate N_R directly based on the mentioned non-linear stress-strain curve. Note that also the relation $N_R = f_c \cdot b \cdot h \cdot (1 - 2e/h)$ suggested in (Schikora and Thomée 2005) should only be applied using design values for the compressive strength of concrete – and not with the actual strength of concrete as required in reliability analysis.

The limit-state function is continuous but not differentiable, as the overall limit-state function is taken as the minimum of all local limit-state functions, corresponding to each cross section of the tunnel lining. For the problem at hand, the shotcrete lining is modeled using 164 beam elements, and a single evaluation of the overall limit-state function requires evaluation of 328 local limit-state functions (i.e., for two cross-sections per beam element).

5.4 Modeling uncertainties

Any engineering model is only an imperfect representation of reality. For the assessed problem, the main causes for modeling uncertainties include: 1) The employed two-dimensional finite element model can only approximately represent three-dimensional effects. 2) The behavior of soil under different stress situations is a very complex problem that can only be approximately represented by the employed material model. 3) The properties of soil are subject to spatial variability. 4) The structural safety of the shotcrete lining is investigated for the material properties of shotcrete after hardening. In reality, the shotcrete is not loaded instantly, but gradually. In tunneling, this is a very complex process, as the shotcrete takes already stresses when it is still hardening. 5) Creep and shrinkage of concrete are not considered in the mechanical model. 6) It is not feasible to model all quantities that are subject to uncertainty. Often only the parameters with the largest uncertainty and the largest impact can be modeled stochastically. The influence of some quantities is actually readily neglected in the engineering model (e.g., we usually model the structure at the macroscopic level and do not explicitly consider what happens at smaller scales).

The relaxation factor α is used to account for the missing three-dimensional effects in the two-dimensional model. By considering the value of α as uncertain, we can – at least partially – account for modeling uncertainties. We model the relaxation factor α as a beta distributed random variable on the interval $[0,1]$ that has a mean of 0.4 and coefficient of variation 10%.

In general, modeling uncertainties must be considered in the analysis (Eurocode 0). However, we here choose to neglect them, because the modeling uncertainty (which leads to an increased probability of failure) is smaller than the degree of conservativeness of the model (which leads to a reduced probability of failure). Neglecting the former partially compensates for the effect of the latter on the estimated reliability.

The model at hand is conservative in the following aspects: 1) Reinforcement in the shotcrete lining is neglected in the limit-state function. 2) The limit-state function also neglects that the concrete can actually take tensile stresses up to a certain amount. 3) Exceeding the admissible moment for a given normal force does usually not lead to failure of the tunnel lining, as assumed in the limit-state function. Instead, the tunnel lining has the potential to redistribute bending moments and to form joints (Schikora and Thomée 2005). 4) The parameter f_{co} that models the stiffness of shotcrete was chosen conservatively (as is discussed in Section 5.2). 5) The chosen probabilistic model for the Young's modulus of shotcrete is on the conservative side (as is discussed in Section 5.2) – i.e., the internal forces in the shotcrete lining are on average larger than to be expected in reality.

6 Reliability analysis of the tunnel

6.1 Representation of the stochastic model in RELY

In a first step, the random variables are defined in the input-file of RELY. This step is illustrated in *Fig. 5*. The definition starts with the keyword VAR and is followed by the name of the random variable. In the study at hand, only the lognormal distribution (LOGN) and the beta distribution (BETA) are applied. However, the list of distribution types that are supported in RELY is comprehensive (SOFiSTiK AG 2016). The random variables listed in *Fig. 5* are defined in terms of their mean (P1) and standard deviation (P2). For the beta distribution, the upper and lower bound of the support of the random variable (P3 and P4) must be specified as well.

```
VAR 'rv_fco'   TYPE LOGN PTYP MOME  P1 33   P2 33*0.16
VAR 'rv_Yc'    TYPE LOGN PTYP MOME  P1 0.8  P2 0.08
VAR 'rv_YE'    TYPE LOGN PTYP MOME  P1 1.0  P2 0.15
VAR 'rv_alpha' TYPE BETA PTYP MOME  P1 0.4  P2 0.04      P3 0  P4 1
VAR 'rv_pref1' TYPE LOGN PTYP MOME  P1 167  P2 55
VAR 'rv_pref2' TYPE LOGN PTYP MOME  P1 167  P2 55
VAR 'rv_pref3' TYPE LOGN PTYP MOME  P1 167  P2 55
VAR 'rv_pref4' TYPE LOGN PTYP MOME  P1 167  P2 55
VAR 'rv_pref5' TYPE LOGN PTYP MOME  P1 167  P2 55
VAR 'rv_pref6' TYPE LOGN PTYP MOME  P1 167  P2 55
VAR 'rv_pref7' TYPE LOGN PTYP MOME  P1 167  P2 55
VAR 'rv_m1'    TYPE LOGN PTYP MOME  P1 0.47 P2 0.47*0.11
VAR 'rv_m2'    TYPE LOGN PTYP MOME  P1 0.47 P2 0.47*0.11
VAR 'rv_m3'    TYPE LOGN PTYP MOME  P1 0.47 P2 0.47*0.11
VAR 'rv_m4'    TYPE LOGN PTYP MOME  P1 0.47 P2 0.47*0.11
VAR 'rv_m5'    TYPE LOGN PTYP MOME  P1 0.47 P2 0.47*0.11
VAR 'rv_m6'    TYPE LOGN PTYP MOME  P1 0.47 P2 0.47*0.11
VAR 'rv_m7'    TYPE LOGN PTYP MOME  P1 0.47 P2 0.47*0.11
VAR 'rv_K04'   TYPE LOGN PTYP MOME  P1 0.68 P2 0.068
VAR 'rv_K05'   TYPE LOGN PTYP MOME  P1 0.49 P2 0.064
VAR 'rv_K06'   TYPE LOGN PTYP MOME  P1 0.49 P2 0.064
VAR 'rv_K07'   TYPE LOGN PTYP MOME  P1 0.68 P2 0.068
```

Fig. 5: Definition of random variables in RELY.

```
let#rho -0.7
CORR 'rv_pref1' 'rv_m1'  #rho
CORR 'rv_pref2' 'rv_m2'  #rho
CORR 'rv_pref3' 'rv_m3'  #rho
CORR 'rv_pref4' 'rv_m4'  #rho
CORR 'rv_pref5' 'rv_m5'  #rho
CORR 'rv_pref6' 'rv_m6'  #rho
CORR 'rv_pref7' 'rv_m7'  #rho
CORR 'rv_K04'   'rv_K05' 0.4
CORR 'rv_K05'   'rv_K06' 0.4
CORR 'rv_K06'   'rv_K07' 0.4
CORR 'rv_K04'   'rv_K06' 0.2
CORR 'rv_K05'   'rv_K07' 0.2
CORR 'rv_K04'   'rv_K07' 0.1
```

Fig. 6: Definition of the correlation between the random variables defined in *Fig. 5*.

The definition of the correlation between the random variables is shown in *Fig. 6*. The correlation between two random variables is specified using the keyword CORR followed by the names of the two random variables and the corresponding correlation coefficient.

The coupling of the stochastic model in RELY to an existing finite element model is depicted in *Fig. 7*. The actual coupling is done using the keyword PROJ. In *Fig. 7*, the SOFiSTiK finite element model is defined in a dat-file that has name 'tunnel_fem.dat'. At the end of the dat-file 'tunnel_fem.dat', the variable 'rel_fun' is stored in the SOFiSTiK database CDB. The variable 'rel_fun' contains the current value of the limit-state function that was evaluated based on the results of the finite element analysis.

Finally, the reliability analysis can be started. To start a FORM analysis, the keyword FORM is used.

```
PROJ dat 'tunnel_fem'
FUNC rel_fun
```

Fig. 7: Coupling of RELY to an existing finite element model.

6.2 Comments on the employed reliability method: FORM

The efficiency of a reliability method is usually expressed in terms of the number of limit-state function evaluations required to obtain an estimate for the probability of failure. The number of limit-state function calls should ideally be small, as each time the limit-state function is evaluated, a non-linear finite element analysis of the mechanical tunnel model must be performed. The efficiency of FORM decreases with an increasing number of random variables in the problem. The problem at hand consists of 22 random variables. For this number of random variables, FORM is still efficient compared to other reliability methods. Based on the design point computed with FORM, importance sampling based reliability methods can be employed to improve the estimated probability of failure.

Besides the assessment of the reliability of the shotcrete lining, we also intend to identify the random variables that have the largest influence on the probability of failure. This is another reason why we employ FORM, as this method provides answers to both questions.

FORM solves a constrained optimization problem. Most optimization algorithms used in combination with FORM require the limit-state function to be differentiable. The gradient of the limit-state function is typically evaluated using a finite difference scheme. For the problem at hand, the employed limit-state function is not differentiable for the following reasons: 1) The limit-state function is expressed as the minimum of local limit-state functions. 2) The solution of the non-linear finite element problem is subject to numerical noise. In principle, the numerical noise of the finite element solver can be reduced, however, this would increase the computational costs of evaluating the limit-state function. In some cases, convergence of the optimization algorithm can be achieved for non-differentiable limit-state functions by increasing the step size of the finite difference scheme.

6.3 Results of the reliability analysis

For the problem at hand, an approximate design point, reflecting the most probable combination of parameters that will lead to failure, can be obtained. The coordinates of the design point as well as the relative importance of the individual random variables are listed in Table 4. Based on the obtained design point and a linearization of the limit-state function around the design point, the reliability index of the tunnel section can be approximated as 4.6; which corresponds to a probability of failure of $2 \cdot 10^{-6}$. The computed reliability index fulfills the demands for safety specified in Eurocode 0. However, the interpretation of the computed reliability is not straightforward, as we investigated only a single section of the tunnel. For a proper assessment of the reliability of the entire tunnel, different tunnel sections must be investigated, to account for the variability of the soil layers and soil properties along the tunnel axis.

The results presented in Table 4 clearly indicate that the reliability is mainly influenced by two random variables: Y_E and $p_{\mathrm{ref,5}}$. This means that the Young's modulus of concrete (through random variable Y_E) and the soil properties of the 5th layer (through quantity $p_{\mathrm{ref,5}}$) have a large influence on the analysis. These two quantities are discussed in the following.

Young's modulus of concrete: The chosen probabilistic description of the Young's modulus of concrete has the largest impact on the reliability. The Young's modulus E_c depends on the two random variables Y_E and f_c; i.e., $E_c = 11 \cdot 10^3 \cdot {f_c}^{0.3} \cdot Y_E$. The random variable f_c is set to the power of 0.3. The expectation and coefficient of variation of ${f_c}^{0.3}$ are 2.66 and 5.7%, respectively. Consequently, the uncertainty on E_c (the coefficient of variation of E_c is $\delta_{E_c} = 16\%$) is dominated by the uncertainty on Y_E (the coefficient of variation of Y_E is $\delta_{Y_E} = 15\%$). Therefore, the relative importance of Y_E is much larger than the importance of $f_c = f_{co} \cdot Y_c$. Failure occurs for large values of Y_E, because in the underlying mechanical model a stiffer lining leads to larger internal forces in the lining. This effect is more pronounced than the decrease of capacity due to a decrease in f_c.

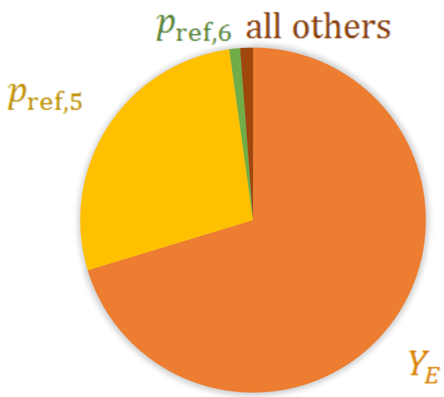

Fig. 8: Relative importance of the stochastic model parameters with respect to the reliability of the tunnel lining.

The result of the high relative importance of the Young's modulus requires careful interpretation. In the selected mechanical model of the tunnel, the material properties of concrete after hardening are used in the analysis. In reality, the shotcrete is loaded gradually. Thus, the speed of tunnel excavation has an influence on the stresses in the tunnel lining. The purpose of the relaxation factor α is to account for this effect. However, the relaxation method is a simplified technique that approximates a highly complex three-dimensional effect. In the employed engineering model, the Young's modulus controls the relative stiffness of the tunnel lining. The relative stiffness of the lining is influenced by many quantities that are due to their complexity not explicitly represented in the engineering model: e.g., the early and gradually increasing loading of the tunnel lining, the temporal progress of hardening, creep and shrinkage. The influence of all these quantities on the effective stiffness of the tunnel lining is implicitly represented by means of the (effective) Young's modulus. Considering the large relative importance of Y_E, detailed (in-situ) studies to improve our knowledge about appropriate values for the Young's modulus of shotcrete in the tunnel lining would be beneficial.

Soil properties: It is not surprising that the properties of soil at the 5th layer have a large influence, as the tunnel is located mainly in this layer. Failure is more likely for large values of $p_{\mathrm{ref},5}$, i.e., for soft soil. For the soil layers located below the tunnel, larger values of p_{ref} are also unfavorable. For all layers located above the 5th soil layer, larger values of p_{ref} are favorable. Note that due to the large relative importance of $p_{\mathrm{ref},5}$, the obtained reliability index is sensitive to the probabilistic description of $p_{\mathrm{ref},5}$. However, the available information about the soil parameters p_{ref} and m is vague (Vogt et al. 2015). Additional information about these parameters would be helpful. The influence of the uncertainty on the value of the lateral earth pressure K_0 on the reliability is negligible.

parameter	mean	x^*	α^2	$\alpha^2_{\mathrm{settlement}}$
f_{co}	$33\mathrm{N/mm}^2$	$32\mathrm{N/mm}^2$	0.2%	0.1%
Y_c	0.8	0.8	0.1%	0%
Y_E	1.0	1.8	70.3%	1%
α	0.4	0.4	0.2%	69.8%
$p_{\mathrm{ref},1}$	167 kN/m²	158 kN/m²	0%	0%
$p_{\mathrm{ref},2}$	167 kN/m²	160 kN/m²	0%	0%
$p_{\mathrm{ref},3}$	167 kN/m²	160 kN/m²	0%	2.7%
$p_{\mathrm{ref},4}$	167 kN/m²	159 kN/m²	0%	1.7%
$p_{\mathrm{ref},5}$	167 kN/m²	337 kN/m²	27.5%	20.3%
$p_{\mathrm{ref},6}$	167 kN/m²	187 kN/m²	1%	0%
$p_{\mathrm{ref},7}$	167 kN/m²	180 kN/m²	0.6%	4.4%
m_1	0.47	0.47	0%	0%
m_2	0.47	0.47	0%	0%
m_3	0.47	0.47	0%	0%
m_4	0.47	0.47	0%	0%
m_5	0.47	0.39	0.2%	0%
m_6	0.47	0.45	0%	0%
m_7	0.47	0.45	0%	0%
$K_{0,4}$	0.68	0.68	0%	0%
$K_{0,5}$	0.49	0.49	0%	0%
$K_{0,6}$	0.49	0.49	0%	0%
$K_{0,7}$	0.68	0.68	0%	0%

Table 4: The coordinates of the approximate design point are listed in column x^*. For comparison, the mean values of the random variables are listed as well. The relative importance of the individual random variables with respect to failure of the tunnel lining is listed in column α^2. The last column lists the relative importance of the random variables with respect to a settlement analysis (see Section 6.3). The random variables with the largest influence are highlighted in *red*.

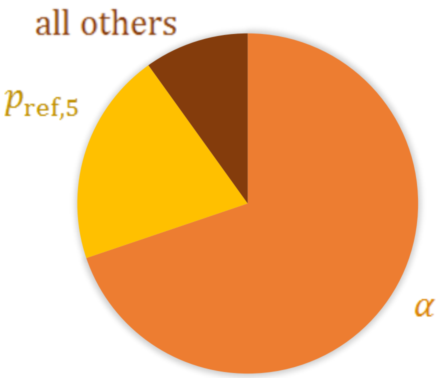

Fig. 9: Relative importance of the stochastic model parameters in a settlement analysis.

6.4 Settlement analysis

In a second study we assess the sensitivity of the stochastic model parameters with respect to settlements at ground level. More precisely, we investigate vertical misalignment of the existing structure depicted in Fig. 4. The system state is considered inadequate if the inclination due to tunnel construction exceeds 1/300. The inclination of the existing structure is evaluated using the vertical settlement at both ends of the structure. More advanced failure scenarios are discussed in e.g. (Meschke 2014).

The sensitivities are computed by means of FORM. The relative importance of the random variables is listed in the last column of Table 4 and depicted in Fig. 9. In this case, the two most important random variables are α and $p_{\mathrm{ref},5}$. Contrary to the investigation of the tunnel lining, the material properties of shotcrete are of minor relevance if ground settlements are investigated. The parameter $p_{\mathrm{ref},5}$ is equally important in both investigations.

The high importance of the relaxation factor α indicates that the settlement analysis is sensitive to changes of this parameter. A three-dimensional finite element model of the tunnel could be beneficial, as it renders the relaxation factor α redundant. However, note that the parameter α is used in the two-dimensional model to implicitly account for modeling errors (compare Section 5.4). It is important to consider modeling errors also in a three dimensional model.

7 Summary

Reliability analysis of engineering structures can help to better understand the behavior of the employed mechanical model. By means of the first-order reliability method (FORM), one can gain useful insights beyond the reliability index (or probability of failure). In particular, FORM provides information on the sensitivity of the reliability to the input uncertainties, which can help in improving the engineering model.

For the considered engineering project of a conventional driven tunnel, we show how to apply the SOFiSTiK module RELY to perform reliability analysis. We mainly concentrate on the reliability of the tunnel lining. FORM is used to approximate the reliability index of the investigated tunnel section, and to determine the stochastic model parameters with the largest influence on the tunnel's reliability. In particular, the effective stiffness of shotcrete was found to have a dominant impact on the reliability of the tunnel lining. Additional studies aimed at improving our knowledge about the effective stiffness of the tunnel lining would be beneficial.

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